Teaching ideas for Option H, *Relativity*

Questions

A number of worksheets are provided for this Option:

- support questions examine the very basic concepts of the syllabus
- extended questions delve deeper and are equivalent to exam level questions.

Teaching ideas

- It is worthwhile spending some time at the beginning of this option carefully discussing the concept of a reference frame and the role of the observer in that frame. It should be stressed that in reality we are talking about an infinite number of observers at each point of space, each with his/her own clock and that all the clocks are synchronised.
- The time of an event is recorded by the observer at the position of that event.
- The idea of a proper interval is also worth spending time on, giving many examples. (Avoid references to ‘observers being at rest relative to events’.)
- The principle of equivalence is basic so it should be used to *deduce* time dilation, light bending and gravitational redshift.
- However, one can also apply the equivalence principle to more mundane topic such as balloons inside accelerating cars, flames on rotating platforms, etc. and ask students to predict the results.

The twin paradox

The twin paradox always creates confusion. Here is a simple quantitative approach to the problem.

The twin paradox is usually described as follows.

An observer, called Rocket, leaves earth towards a distant solar system. Once Rocket arrives at the solar system he immediately turns around to head back towards earth. The ‘paradox’ arises when we consider the rocket observer’s twin sister, Earth, who has stayed behind. Earth states that she is at rest and so claims that Rocket must be younger than Earth when Rocket returns. This is because of time dilation and the fact that ‘moving clocks run slow’. However, Rocket claims that it is he who is at rest and that it was Earth who moved away and then turned around to meet with Rocket again. In this way, Earth must be younger than Rocket since ‘moving clocks run slow’.

Since both cannot be right, we have a ‘paradox’.

Let us choose some convenient numbers in order to make the arithmetic simple. Let Rocket move away at speed v = 0.8c and let the total journey take 30 years (15 years out and 15 years in) for Rocket. Earth would claim that she herself aged by $\gamma \times 30 = \frac{5}{3} \times 30 = 50$ years.

If we take Rocket’s point of view, we would expect that if he aged by 30 years then Earth would age by $\frac{30}{\gamma} = \frac{30}{5/3} = 18$ years.

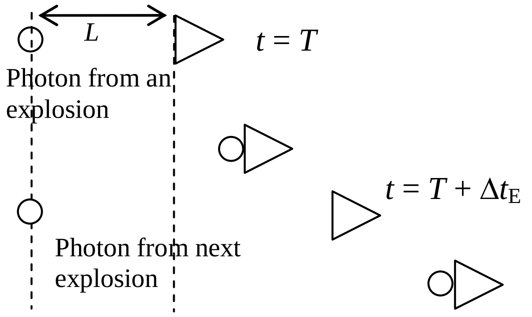
Who is right?

In order to analyse the problem and see who, in fact, is the younger of the two observers when they are reunited, we imagine that the earth observer sets up huge explosions of fireworks on her birthdays. The rocket observer can see the fireworks (using super telescopes!) and so can keep a record of her twin’s birthdays.

Suppose then that Rocket moves away at speed v. How often, according to rocket clocks, are the fireworks seen by Rocket? The fireworks are set off every Δt_E = 1 year according to Earth. Notice that the earth and the rocket observers have synchronised clocks, that is to say as the rocket passed over earth, clocks at the origin of both frames were set to show zero.

On the outward journey of the rocket, let L be the distance separating earth and the rocket on a birthday of the earth observer, according to Earth.

The diagram shows photons emitted at two successive birthdays of Earth. The diagram is from the point of view of Earth.



The photon from an explosion is emitted at time T (according to Earth) and will reach the rocket after a time t_1 given by $ct_1 = L + vt_1 \Rightarrow t_1 = \frac{L}{c-v}$. In other words the photon will arrive at time $T + \frac{L}{c-v}$ since time zero (according to Earth).

The photon from the explosion of the next birthday will be emitted at time $T + \Delta t_{\mathrm{E}}$ (according to Earth). The distance separating earth and rocket then is $L + v\Delta t_{\mathrm{E}}$ (according to Earth). This photon will arrive at the rocket after a time t_2 given by $ct_2 = L + v\Delta t_E + vt_2 \Rightarrow t_2 = \frac{L + v\Delta t_E}{c-v}$, i.e. at a time $t = T + \Delta t_E + \frac{L + v\Delta t_E}{c-v}$ since time zero. The difference between these arrival times is (according to Earth) $T + \Delta t_E + \frac{L + v\Delta t_E}{c-v} - (T + \frac{L}{c-v}) = \Delta t_E + \frac{v\Delta t_E}{c-v} = \frac{c\Delta t_E}{c-v}$

Now the arrival of the two photons occurs at the same place in the rocket frame and so the rocket measures a *proper time interval* Δt_{R} for the time in between these arrivals. Hence

$$\Delta t_R = \frac{\frac{c\Delta t_{\mathrm{E}}}{c-v}}{\gamma} = \frac{c\Delta t_{\mathrm{E}}}{c-v}\sqrt{1-\frac{v^2}{c^2}} = \frac{\Delta t_{\mathrm{E}}}{1-\frac{v}{c}}\sqrt{1-\frac{v^2}{c^2}} = \Delta t_{\mathrm{E}}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} = 1\times\sqrt{\frac{1.8}{0.2}} = 3 \text{ years}$$

So Rocket sees flashes every 3 years (according to rocket clocks). The outward journey lasted for 15 years by rocket clocks, and so Rocket will record $\frac{15}{3} = 5$ birthdays.

On the return journey, all we have to do is change the sign of v in the formula above to get

$$\Delta t_R = \frac{\frac{c\Delta t_{\mathrm{E}}}{c+v}}{\gamma} = \frac{c\Delta t_{\mathrm{E}}}{c+v}\sqrt{1-\frac{v^2}{c^2}} = \frac{\Delta t_{\mathrm{E}}}{1+\frac{v}{c}}\sqrt{1-\frac{v^2}{c^2}} = \Delta t_{\mathrm{E}}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} = 1\times\sqrt{\frac{0.2}{1.8}} = \frac{1}{3} \text{ year}$$

Rocket sees flashes every $\frac{1}{3}$ year. The inward journey lasted for 15 years by rocket clocks, and so Rocket will record $\frac{15}{1/3} = 45$ birthdays.

In total, 30 years have gone by for the rocket observer, Rocket has aged by 30 years, but 5 + 45 = 50 years have gone by for the earth-bound twin, Earth, so she has aged by 50 years.
This shows that it is the earth bound-twin's point of view that is correct.

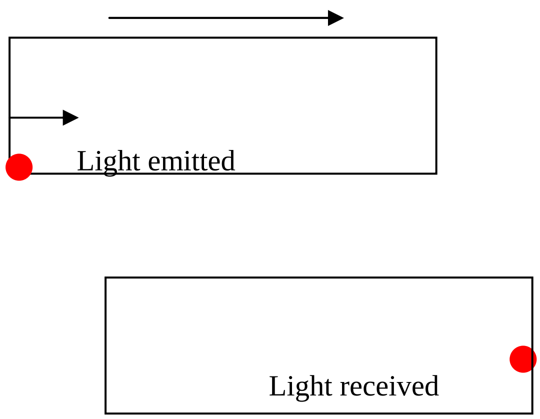
Practical activities/ICT

There can be very few practical activities in this option! By way of activities, consider the following:

- Students can be asked to investigate whether Einstein was aware of the Michelson–Morley experiment, and, if he was, to what extent, if at all, he was influenced by it.
- The following site from the University of South New Wales in Australia http://www.phys.unsw.edu.au/einsteinlight/ (last visited February 2012) has interesting discussions, demonstrations, explanations and animations on all aspects of the theory of relativity.
- The 2008 BBC production 'Einstein and Eddington' (BBC DVD, 90 minutes long) is well worth showing to any relativity class!

Common problems

- Students often have problems with the concept of proper time interval and erroneously assume that proper time is always measured by the 'moving observer'. Consider a train moving past a platform. A man sitting on a bench on the platform starts talking on his cell phone as a train moves past the platform and a few minutes later ends the conversation. Who measures a proper time interval between the time the conversation starts and the time the conversation stops – the platform-based observer or the train observer?
- Consider light emitted from the back wall inside the train that moves to the right relative to the ground. The light is received at the front of the train. Discuss why neither the train observer nor the ground observer measures a proper time interval between the emission of the light at the back wall and its arrival at the front wall.



Theory of knowledge (TOK)

The theory of relativity is full of opportunities for TOK-related discussions. These could include:

- the nature of space and time and the loss of absolute time
- the connection between physics and mathematics and the development of non-Euclidean geometries – to what degree do the needs of one discipline influence development in the other?

Intuition – the case of John Archibald Wheeler

Intuition plays a great role in physics and in the way physicists discover things. An excellent example of intuition in physics is the case of John Wheeler (1911–2008).

This legendary Princeton physicist is responsible for introducing the term 'black hole' into the glossary of physics. He was also responsible for many other terms, such as the Planck length (the small length below which quantum gravitational effects become important), the Planck time, the wormhole, the geon (an object consisting of just gravitational waves), the quantum foam (space at a very small scale looks more like foam than a smooth surface), the theorem that 'black holes do not have hair' (black holes are only described by the charge, mass and angular momentum – no other characteristic can be measured from the outside) and his famous one-line summary of all of general relativity: 'space tells matter how to move and matter tells space how to curve'.

Wheeler, whose many claims to fame also include being Richard Feynman's Ph.D. advisor, is known for many cases of intuition. Physicists often refer to 'intuition' as a way of getting to the answer of a problem. Intuition may be defined as the abrupt 'jump' to a solution or way to a solution without the (apparent) use of logical inference or experimental work. As such, knowledge derived from intuition alone is suspect. The use of intuition by physicists often involves the arrival at a hint or even a complete solution to a problem, which must then be justified by the usual scientific method. Having got the answer or part of the answer by an 'intuitive leap', the subsequent filling in of the justifying details often becomes easier.

One of the most famous examples in the use of intuition concerns the very existence of black holes. The first solution of Einstein's equations that gave rise to the black hole was provided by the German astronomer Karl Schwarzschild in 1918. However, that solution was regarded as a mathematical curiosity and not representing anything physical until the late 1950s. It was then that Wheeler tried to answer the question of whether a black hole, if formed, is stable against perturbations of various kinds. If it was not, then its physical significance would be worthless. Tackling the problem of black hole stability proved to be, mathematically, exceedingly complex. Wheeler wrote a research paper called 'On the stability of the Schwarzschild singularity' in which no equations appeared! There were conclusions drawn but no equations to support them. Wheeler's intuition guided him to the correct answers. T. Regge, then a graduate student, was introduced to Wheeler as someone brilliant enough to tackle the difficult mathematics of the paper. After a few months of work, Regge filled in the missing equations. Surprisingly, almost all of Wheeler's conclusions were justified by Regge's mathematical derivations!

